CURRICULUM, PEDAGOGY AND BEYOND

THE MATHEMATICAL ASSOCIATION OF VICTORIA

MAV24 CONFERENCE

C27 - Strengthening connections and understanding - linear functions and models

Year 7 – 10 Algebra

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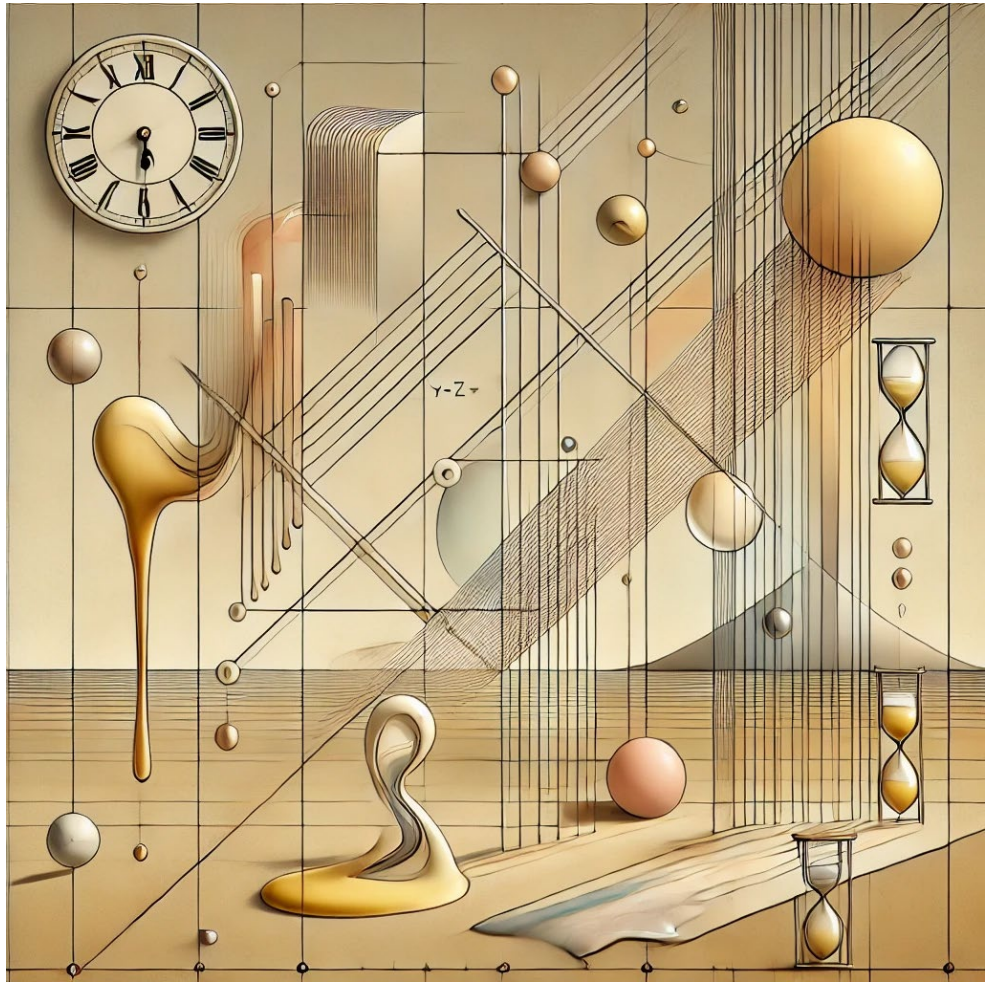
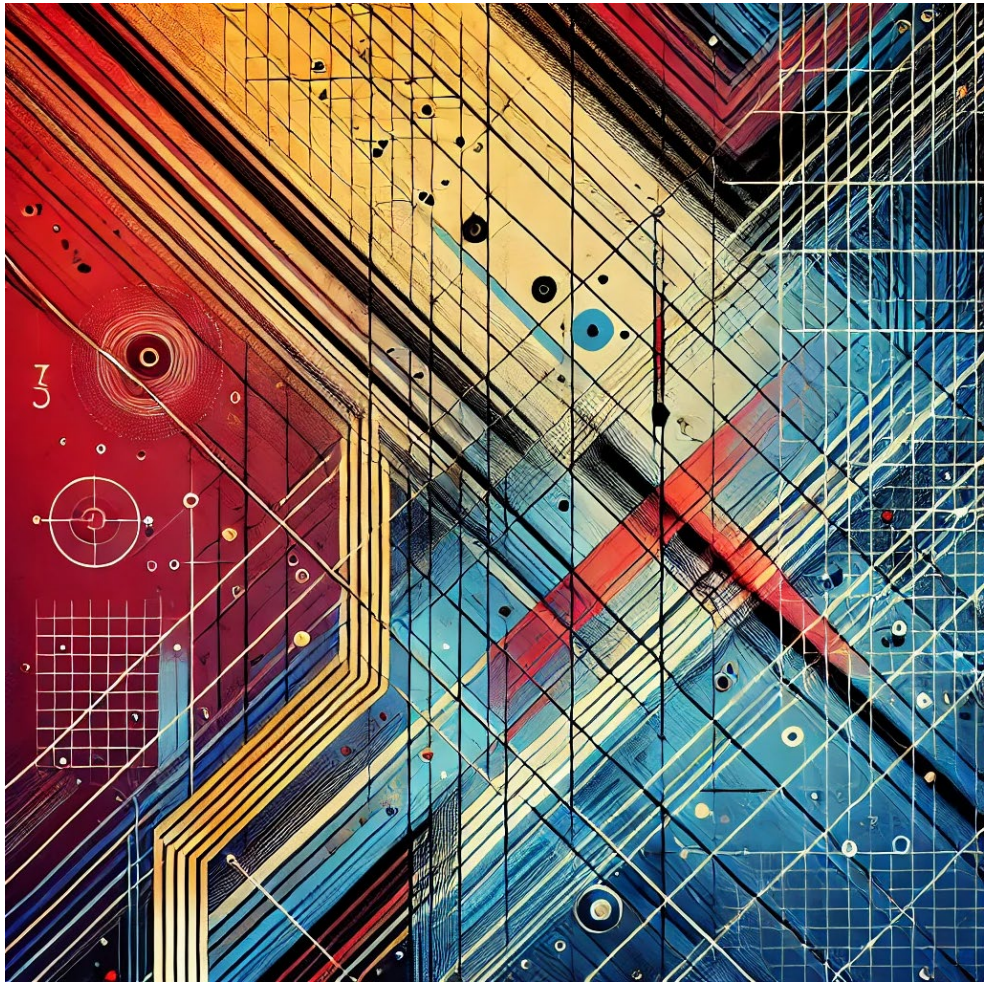




Today's session

Strengthening connections and understanding – linear functions and models

Linear functions are a fundamental type of function studied in detail throughout secondary school, Years 7-10, and yet many learners struggle with various aspects of them even at VCE. How might we address this?

This interactive session will engage participants with a range of activities and approaches that can be used to enable students to progressively build their understanding, make connections and develop their skills in working with linear functions and models from Year 7 through to VCE.

Today's session - sequence

- Overview - key concepts of linear functions
- Some examples of linear function models
- Activity (tables, rules, graphs, equations and models)
- Curriculum mapping for the activity with respect to VC Mathematics 2.0
- Research links and resource links
- Any quick questions?
- Thank you

Overview

Key ideas of linear functions and models

- important background from the primary years is skip counting backwards or forwards by a fixed amount from a given starting number
- modelling contexts: starting value, constant rate of change, domain of application
- constant difference between consecutive values in a table, the output value corresponding to an input of zero
- straight line graph, gradient and vertical axis intercept
- algebraic rule of the form $y = a\,x + b$
- evaluating a linear function or solving linear equations only involves the four basic arithmetic operations, typically with integers, fractions and decimals.
- line segment graphs (for example: tax scales, constant speed motion graphs)

Skip counting

Count backwards or forwards by a fixed amount from a given starting number

- start at 17 and skip count forwards in 3's (is 217 part of this sequence of numbers?)
- start at 17 and skip count backwards in 3's (is -28 part of this sequence of numbers?)
- skip count forwards in $\frac{2}{3}rds$ starting at 5
- skip count backwards in $-0.35\ ths$ starting at 6

Question: given the starting number the size of the amount for skip counting, and the direction (forwards/backwards), how can you tell if a particular number is part of the sequence generated or not?

Deducing a rule from a table of values (1)

Rule is of the form output equals input multiplied by a fixed number, with another fixed number added or subtracted.

Teacher as function machine, students provide possible input values, teacher gives the corresponding output value (teacher may decide not to provide output values for some inputs).

x	*y*
Rule:	

x	*y*
Rule:	

x	*y*
Rule:	

x	*y*
Rule:	

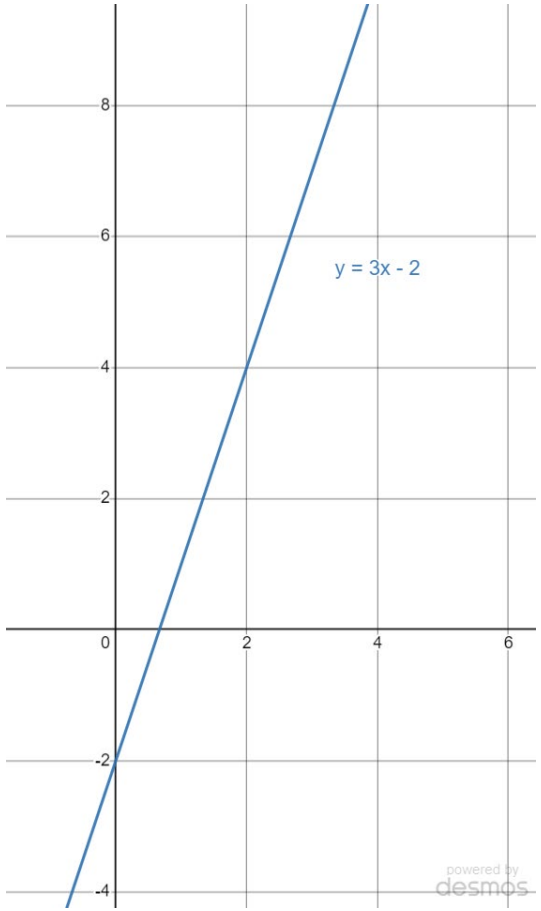
Deducing a rule from a table of values (2)

Teacher as function machine, students to determine the rule

x	y
0	-2
1	1
2	4
3	7
4	10

x	y
4	10
8	
2	4
5	13
	25

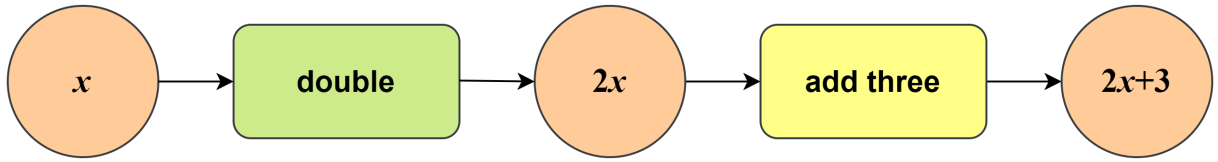
$$y = a\,x + b$$



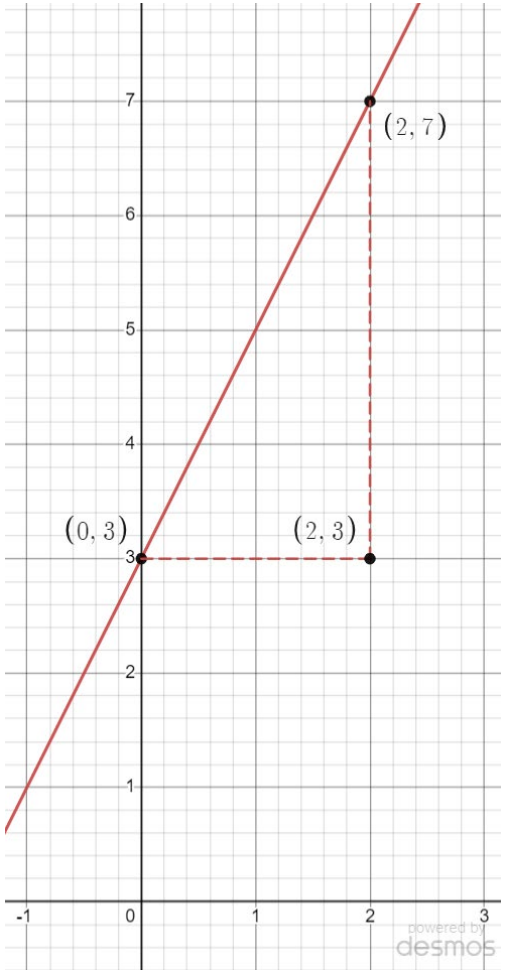
Source: Graph drawn using Desmos

Rule, table and graph

$$y = 2x + 3$$

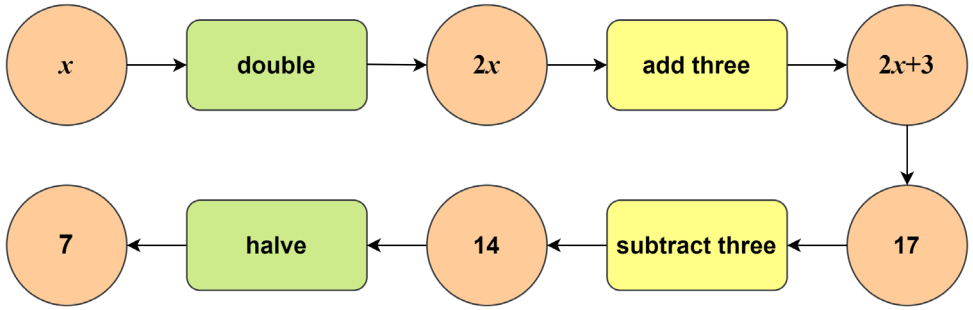


Input (x)	Output (y)	
0	3	> 2
1	5	> 2
2	7	> 2
3	9	> 2
4	11	> 2
5	?	> 2

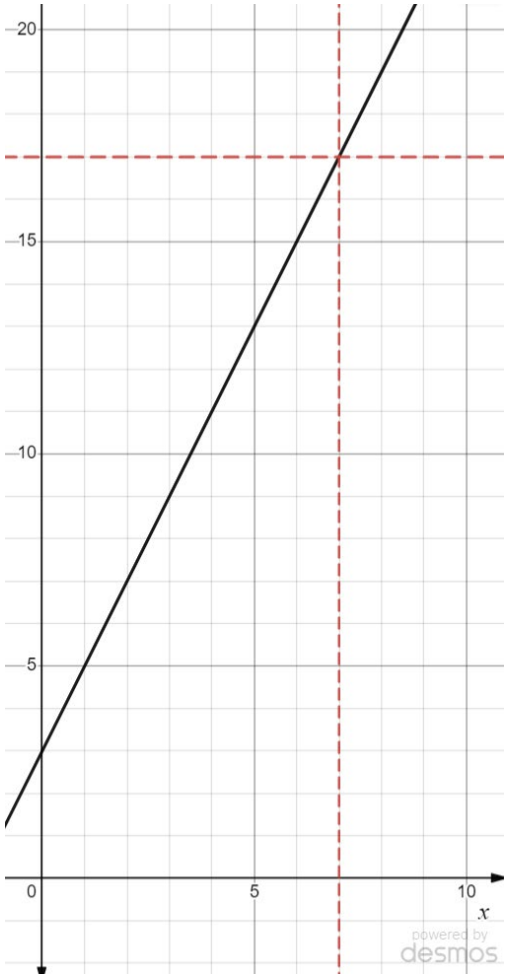


Source: Graph drawn using Desmos

Solving a basic linear equation: $2x + 3 = 17$



$$2x + 3 = 17$$
$$2x + 3 - 3 = 17 - 3$$
$$2x = 14$$
$$\frac{2x}{2} = \frac{14}{2}$$
$$x = 7$$



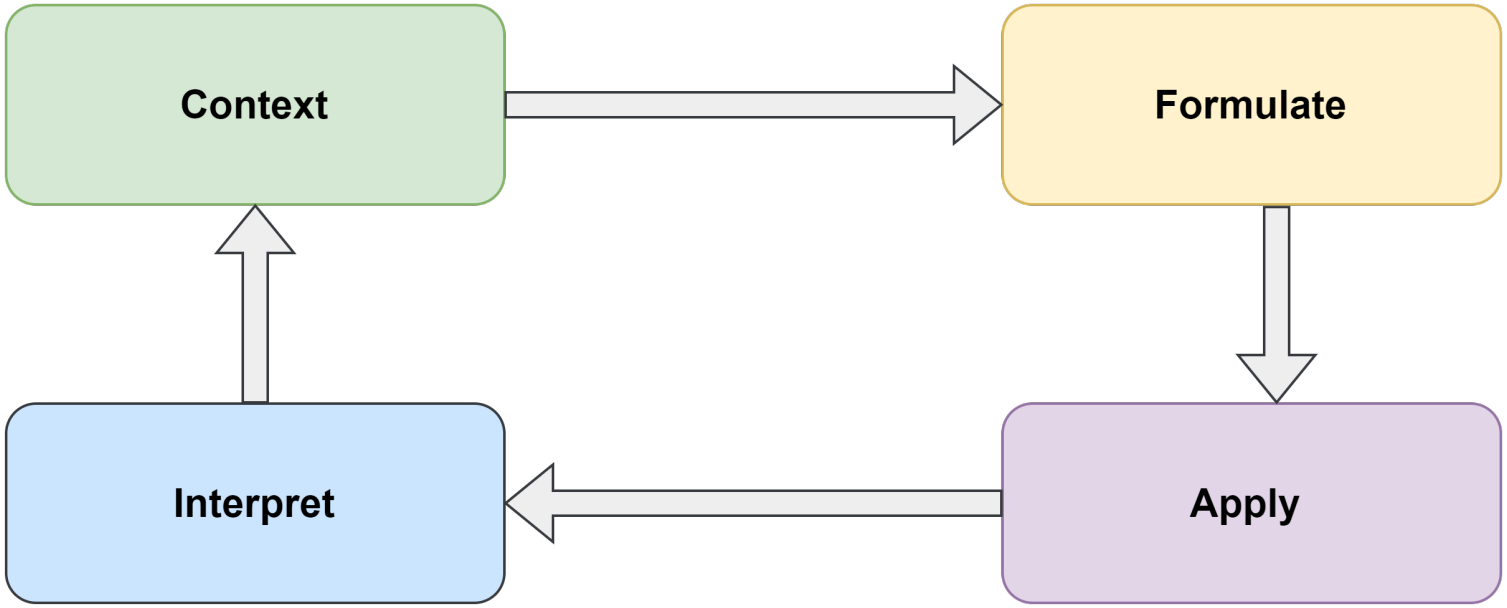
x	$y = 2x + 3$
0	3
1	5
2	7
3	9
4	11
5	13
6	15
7	17
8	19
9	21
10	23

Source: Graph drawn using Desmos

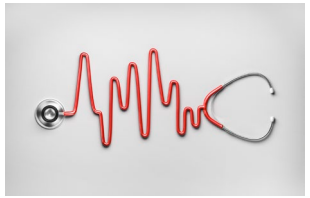
Linear models

Some examples

A mathematical modelling schema



Exercise – max heart rate models by rule and table



Simple linear : Model 1

Maximum heart rate = 220 – 1 × age

More detailed linear : Model 2

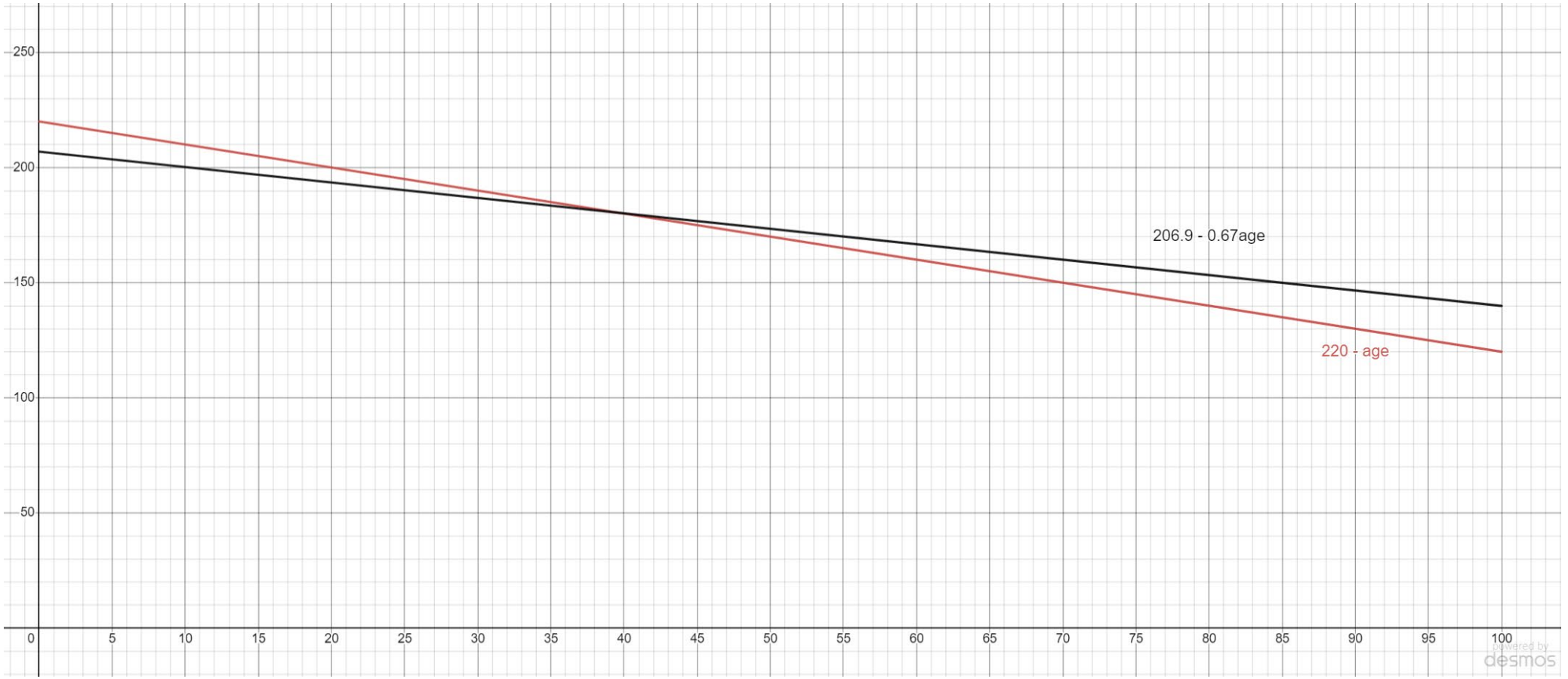
Maximum heart rate = 206.9 – 0.67 x age

Note: max heart rate is often used to work out a 60% – 80% heart rate range for exercise

Source: Calculators | Heart Online

Age	Model 1	Model 2
0	220	206.9
10	210	200.2
20	200	193.5
30	190	186.8
40	180	180.1
50	170	173.4
60	160	166.7
70	150	160
80	140	153.3
90	130	146.6
100	120	139.9

Exercise – max heart rate models by rule and graph



Model 1: Maximum heart rate
= 220 – 1 × age

Model 2: Maximum heart rate
= 206.9 – 0.67 x age

Source: Graph drawn using Desmos

Forensic science

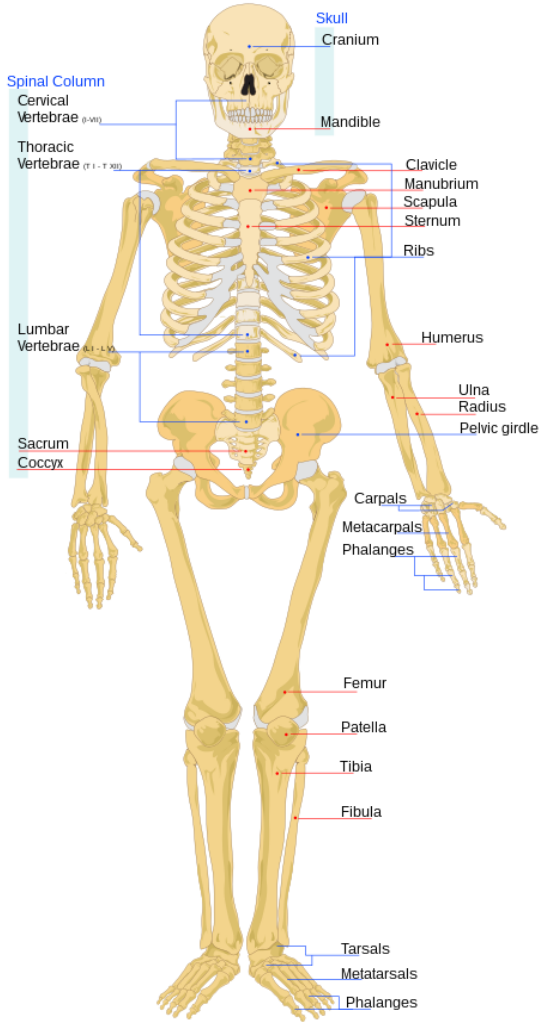
Predicting height from bone length the radius arm bone (elbow to wrist)

Female: $f = 3.9 \times r + 73.41$ cm **Male:** $m = 3.7 \times r + 80.5$ cm

Sia's radius arm bone length is 26 cm, how tall is she?
Mo's radius arm bone length is 28 cm, how tall is he?

What is the predicted radius arm bone length for:

- A 2-metre in height female? A 2-metre in height male?
- Measure your radius arm bone length, what height does this predict? How accurate is the prediction?



Source: File:Human skeleton front en.svg - Wikipedia

Solving an equation by algebra or technology (1)

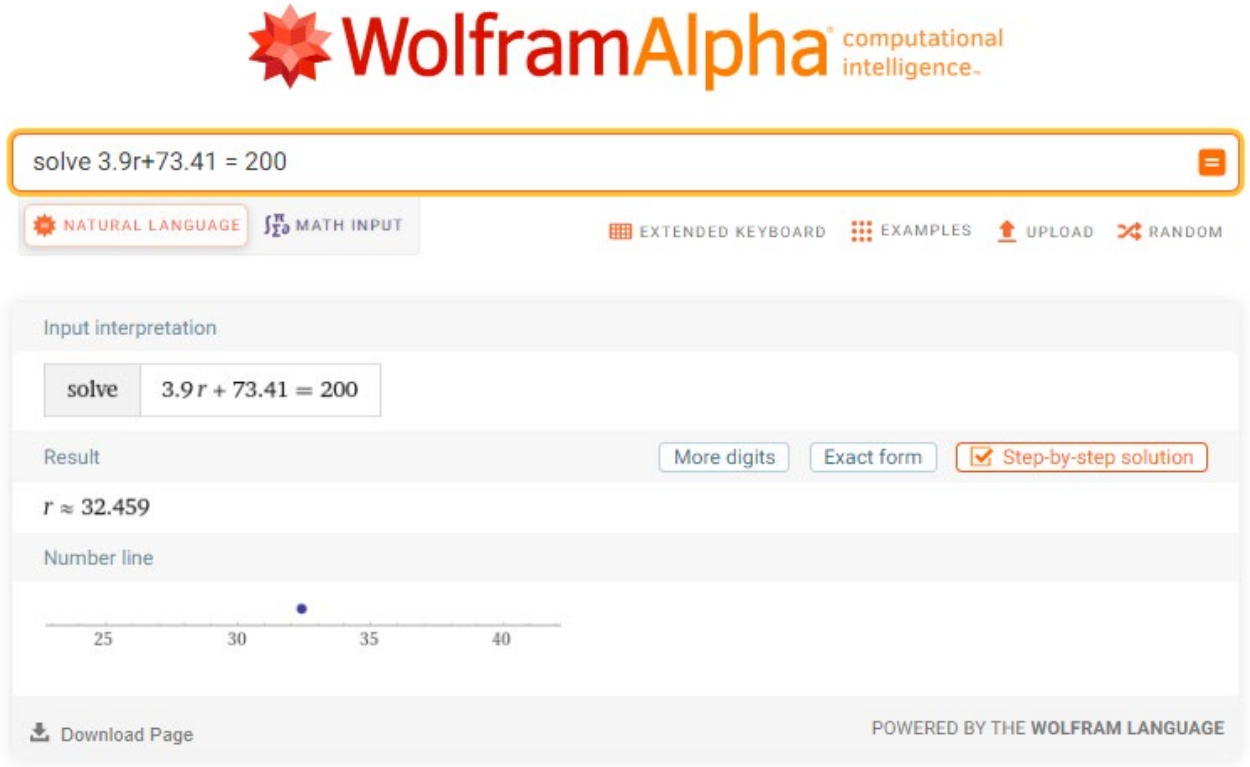
Use a calculator to evaluate the solution:

$$3.9r + 73.41 = 200$$

$$3.9r = 200 - 73.41$$

$$r = \frac{200 - 73.41}{3.9}$$

$$r \approx 32$$



Source: Wolfram|Alpha: Computational Intelligence (wolframalpha.com)

Solving an equation by algebra or technology (2)

Microsoft | Math Solver

$3.9r + 73.41 = 200$

Solve for r

$$r = \frac{12659}{390} = 32\frac{179}{390} \approx 32.458974359$$

View solution steps

[Solve 3.9r+73.41=200 | Microsoft Math Solver]

Hide steps

Steps for Solving Linear Equation

$$3.9r + 73.41 = 200$$

Subtract 73.41 from both sides.

$$3.9r = 200 - 73.41$$

Subtract 73.41 from 200 to get 126.59.

$$3.9r = 126.59$$

Divide both sides by 3.9.

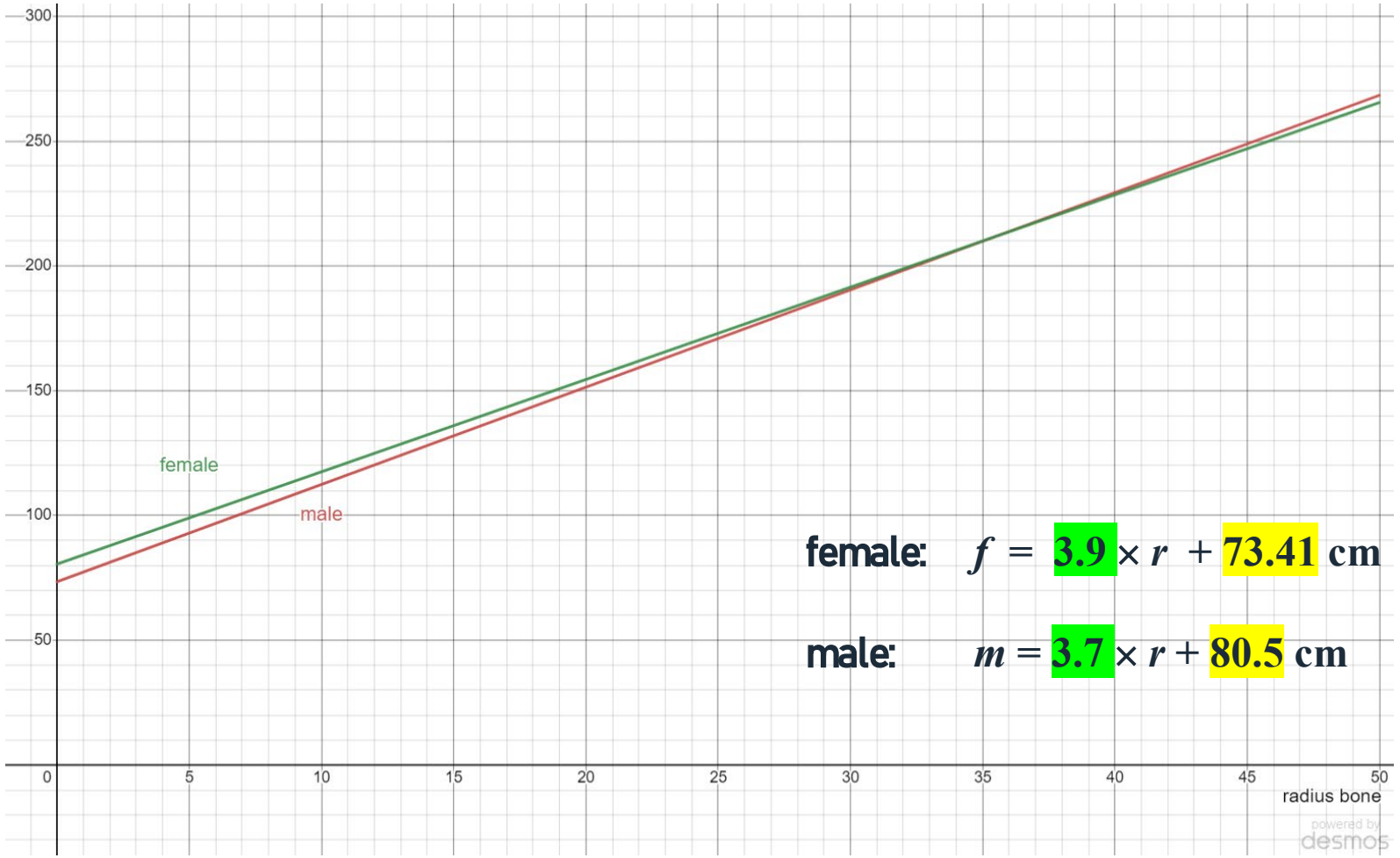
$$r = \frac{126.59}{3.9}$$

Expand $\frac{126.59}{3.9}$ by multiplying both numerator and the denominator by 100.

$$r = \frac{12659}{390}$$

Forensic science

Predicting height from the radius arm bone length (elbow to wrist)



Source: Graph drawn using Desmos

Piecewise linear (line segment) function

Resident tax rates for 2023–2024	
Taxable income	**Tax on this income**
0 – $18,200	Nil
$18,201 – $45,000	19 cents for each $1 over $18,200
$45,001 – $120,000	$5,092 plus 32.5 cents for each $1 over $45,000
$120,001 – $180,000	$29,467 plus 37 cents for each $1 over $120,000
$180,001 and over	$51,667 plus 45 cents for each $1 over $180,000

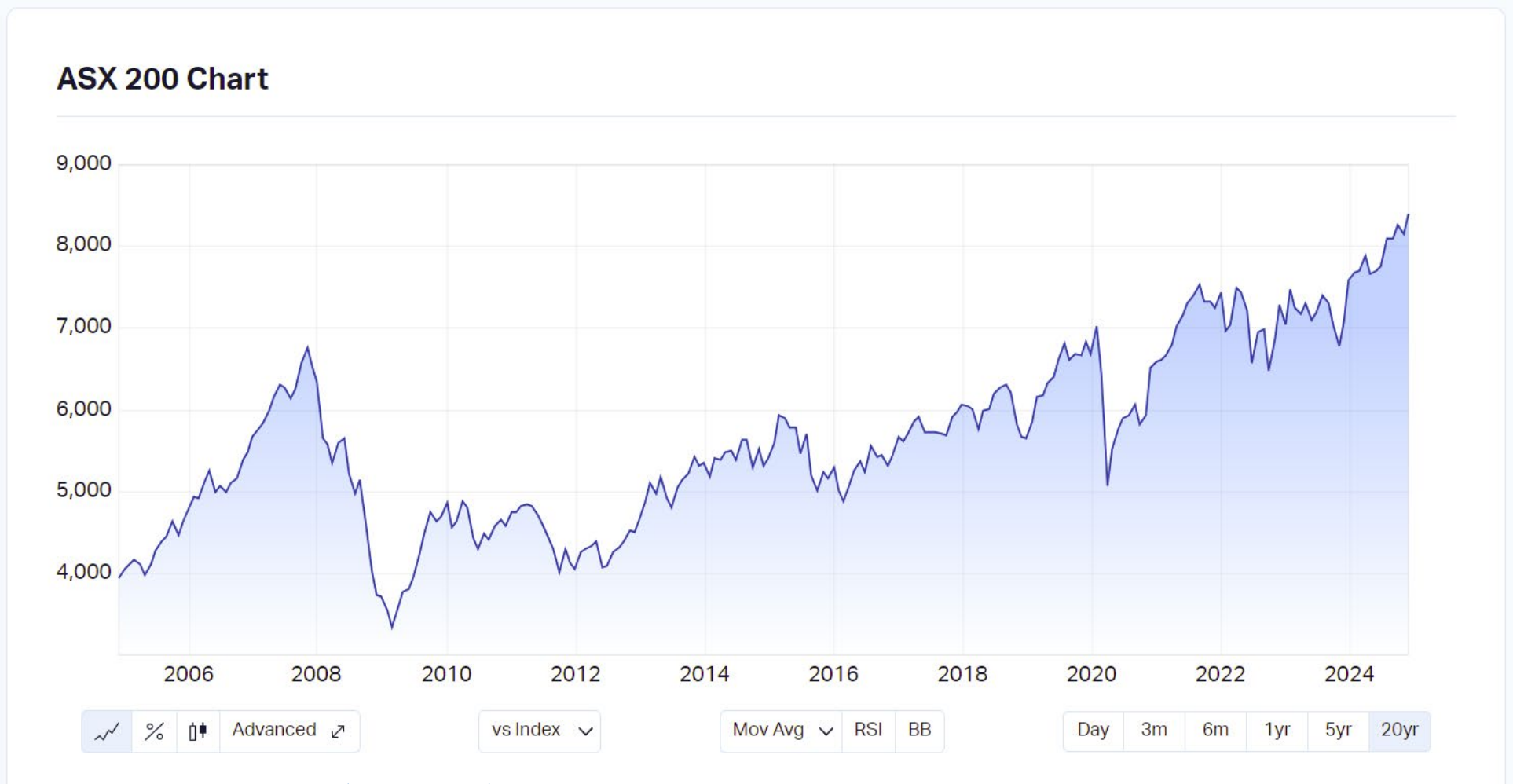
What's the tax on $ 90 000?

Simple tax calculator | Australian Taxation Office

Tax = 5092 + 0.325 × (90000 – 45000)

Source: Tax rates – Australian resident | Australian Taxation Office – does not include Medicare levy.

Piecewise linear (line segment) function



Source: S&P/ASX 200 (LIVE DATA): Share Prices & Charts - Market Index 27 Nov 2024

Overview -recap

A linear function is specified by two pieces of information (parameters):

- an initial (starting) value, b
- the constant rate of change, a
- these are represented **algebraically** by a rule of the form $y = ax + b$

They are represented graphically by the point corresponding to the vertical or y-axis intercept at
$(0, b)$ and the gradient a of the line segment joining any two distinct points on the line, calculated as rise/run.

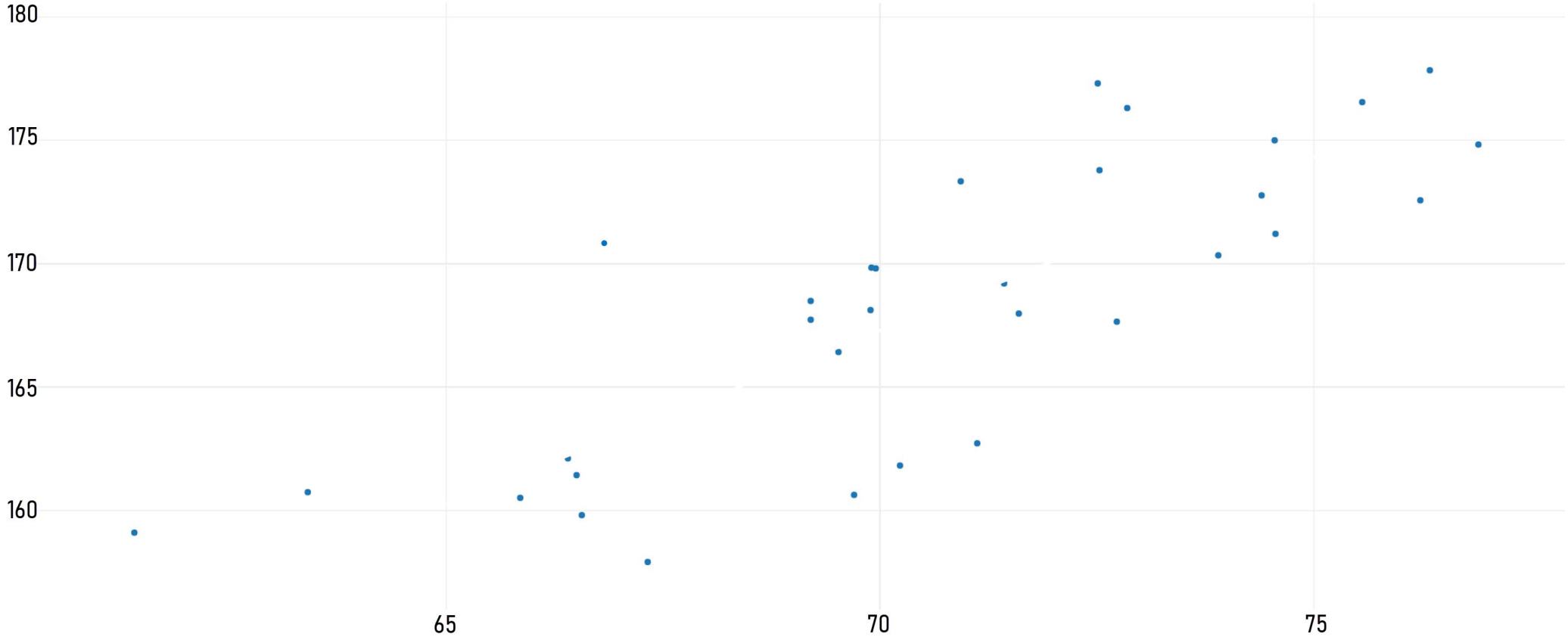
They are represented numerically in a table of values as the output value $y = b$ from the rule corresponding to the input value of $x = 0$; and the common difference of a between output values for consecutive input values.

In **modelling contexts**, the constant rate of change and initial value are interpreted with respect to the relationship between the variables, and the **domain** of the function, that is, the set of input values for which it makes sense to apply the function.

Activity

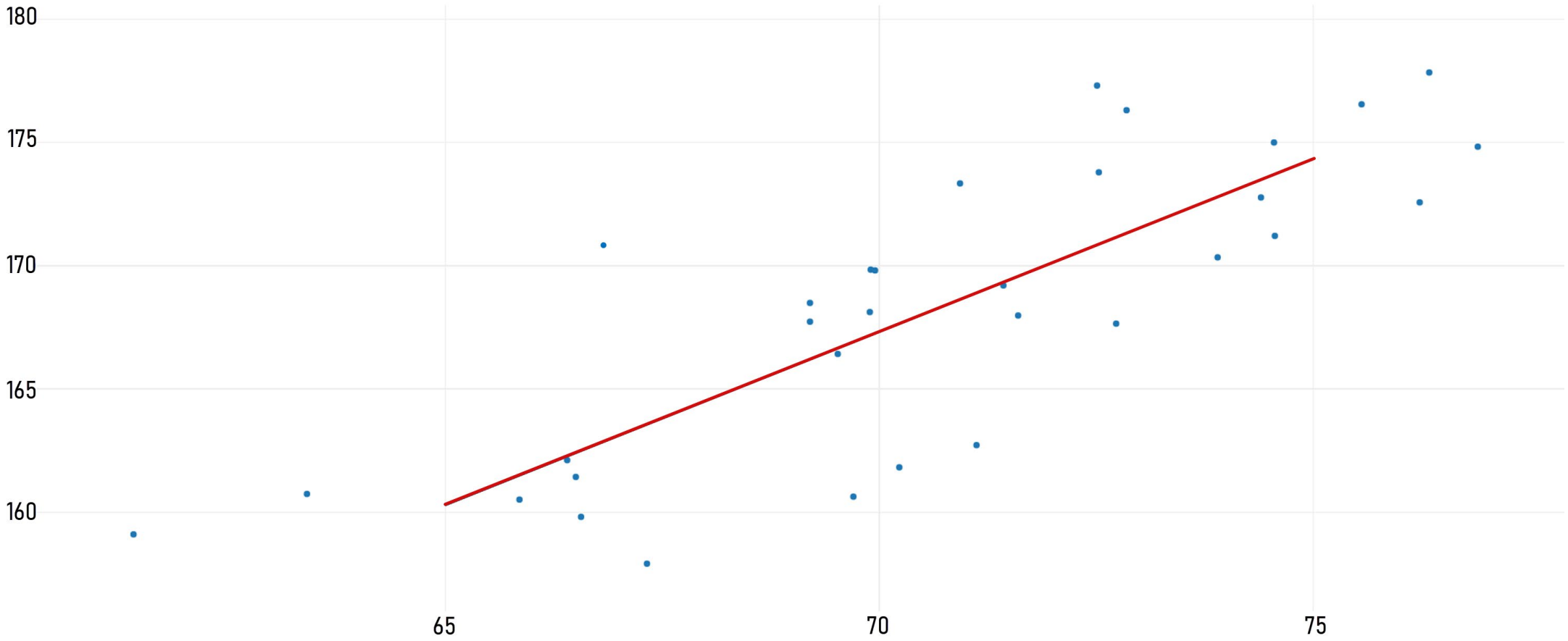
Graphs, rules and models

Forensics – left arm length and height



Source: Height Vs. Left Arm Length | scatter chart made by 16cduncan | plotly

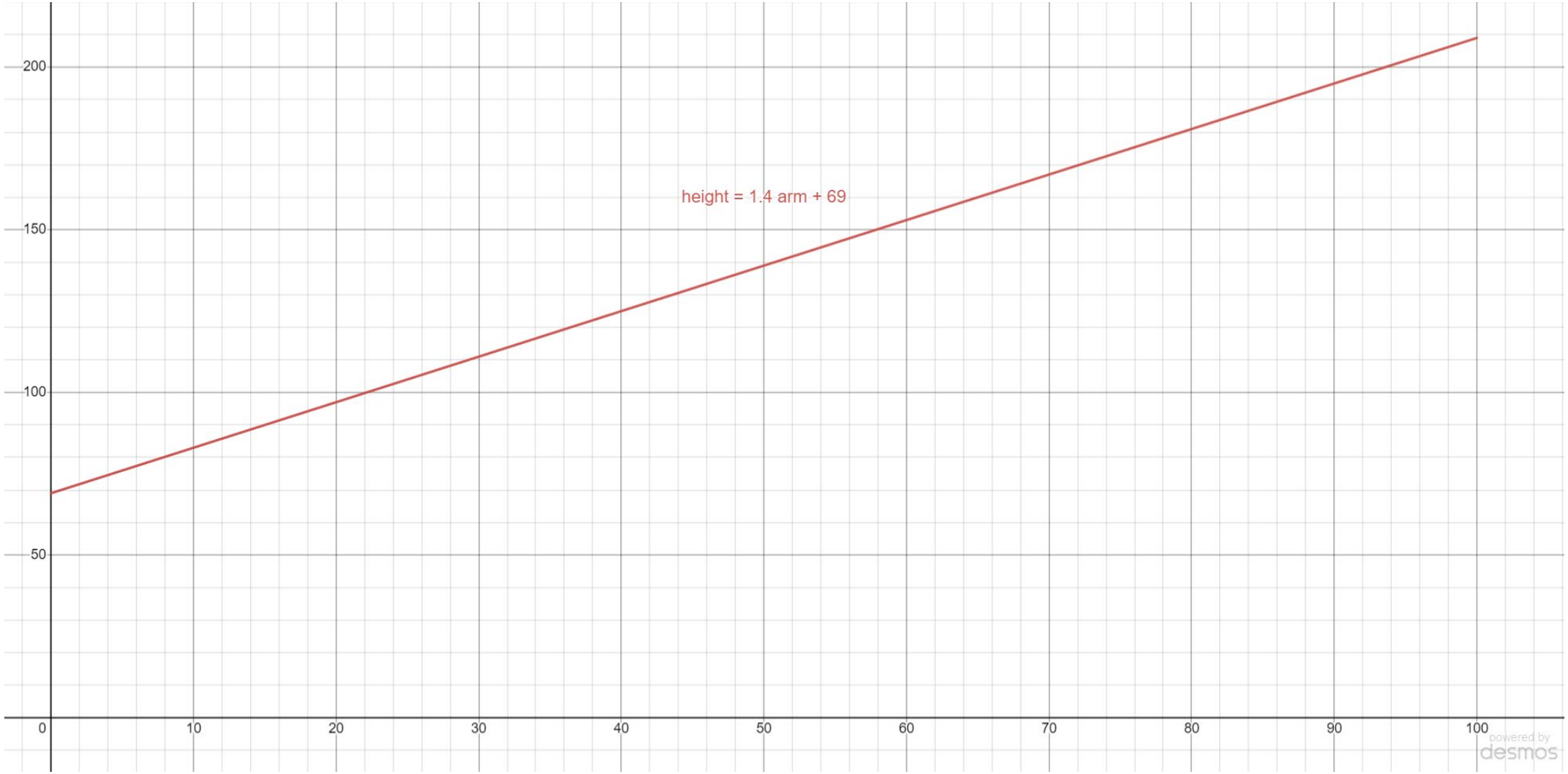
Forensics – left arm length and height



Source: Height Vs. Left Arm Length | scatter chart made by 16cduncan | plotly

What's the gradient? What would the initial value be?

Forensics – left arm length and height



Source: Graph drawn using Desmos

As a whole class activity

Have students construct rules based on their own class data

- gather class data for elbow to middle fingertip length (cm) and height (cm)
- draw a scatterplot of this data and have students find a line of good fit by eye and determine its rule graphically (all students data, male student data and female student data separately)
- discuss approaches students use to find their line of good fit by eye, including consideration of any outliers
- compare predictions from these (for example: measurements for teachers, students from other classes) with previous model rules, calculate percentage error in predictions

Extension

- compare models based on different body lengths
- Develop a common approach to finding a rule

Extension – a common approach

Many students = many ‘by eye’ possible model rules

When students find a modelling rule by fitting a line by eye, there will be a range of rules, and predicted values.

These can vary considerably depending on the gradient, so how could all students obtain a common rule?

The following is a simple possible approach:

- divide the data into two equal size groups (lower half, upper half) based on arm length values
- calculate the average (mean) *x* and *y* value for each group
- use these as coordinates to determine a linear model for the data
- consider the impact of any outliers (means are sensitive to outliers)

A neat way to find the rule – an example

Rule from the coordinates of two points, for example (65, 160) and (75, 174)

Step 1: Let $y = ax + b$, substituting coordinates for the points into this rule gives:

Equation 1: $160 = 65a + b$ and **Equation 2**: $174 = 75a + b$

Step 2: Subtracting **Equation 1** from **Equation 2** and making a the subject gives:

$14 = 10a$ so $a = 1.4$

Step 3: The value of a is now known, so substituting this into, for example, **Equation 1**, and making b the subject gives respectively:

Equation 1: $160 = 65 \times 1.4 + b$ so $b = 160 - 91 = 69$

Rule: $y = 1.4x + 69$ (this is a model rule for the activity just completed based on these two points)

Note that for measurement data in general a calculator or other technology will need to be used to find the values of a and b.

Finding the rule from two points – general approach

Rule from two points $(x_1, y_1), (x_2, y_2)$

Step 1

Let $y = ax + b$, substituting coordinates for the points into this rule gives:

Equation 1: $y_1 = ax_1 + b$ and **Equation 2**: $y_2 = ax_2 + b$

Step 2

Subtracting **Equation 1** from **Equation 2** and making a the subject gives:

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 3

The value of a is now known, so substituting this into either of **Equation 1** or **Equation 2** and making b the subject gives respectively:

$b = y_1 - ax_1$ or $b = y_2 - ax_2$

Note that for measurement data such as this a calculator or other technology will need to be used to find the values of a and b.

Curriculum mapping

Activity and VC: Mathematics V2.0 content descriptions

VC 2.0 linear related content descriptions

Amended_13/11/2024_linear_scope and sequence – Levels 7–10

Strand: Algebra

Level 7	Level 8	Level 9	Level 10
recognise and use variables to represent everyday formulas algebraically and substitute values into formulas to determine an unknown VC2M7A01	create, expand, factorise, rearrange and simplify linear expressions, applying the associative, commutative, identity, distributive and inverse properties VC2M8A01		
apply the associative, commutative and distributive laws to aid mental and written computation, and formulate algebraic expressions using constants, variables, operations and brackets VC2M7A02			
solve one-variable linear equations of increasing complexity with natural number solutions; verify equation solutions by substitution VC2M7A03	graph linear relations on the Cartesian plane using digital tools where appropriate; solve linear equations and one-variable inequalities using graphical and algebraic techniques; verify solutions by substitution VC2M8A02	sketch linear graphs of equations in various algebraic forms, using the coordinates of 2 points, and solve linear equations VC2M9A03	solve problems involving linear equations, including those derived from formulas VC2M10A07
investigate, interpret and describe relationships between variables represented in graphs of functions developed from authentic data VC2M7A04		find the gradient of a line segment, the midpoint of the line interval and the distance between 2 distinct points on the Cartesian plane VC2M9A04	solve linear inequalities and graph their solutions on a number line VC2M10A08
generate tables of values from visually changing patterns or the rule of a function; describe and plot these relationships on the Cartesian plane VC2M7A05			solve simultaneous linear equations, using algebraic and graphical techniques including using digital tools VC2M10A09
manipulate formulas involving several variables using digital tools, and describe the effect of systematic variation in the values of the variables VC2M7A06			solve problems involving gradients of parallel and perpendicular lines VC2M10A10
			solve linear equations involving simple algebraic fractions VC2M10A12
	experiment with linear functions and relations using digital tools, making and testing conjectures and generalising emerging patterns VC2M8A05		
	use mathematical modelling to solve applied problems involving linear relations, including financial contexts involving profit and loss; formulate problems with linear functions, and choose a representation; interpret and communicate solutions in terms of the context, and review the appropriateness of the model VC2M8A03	use mathematical modelling to solve applied problems involving change, including financial contexts involving simple interest; formulate problems, choosing to use either linear or quadratic functions or other simple variations; interpret solutions in terms of the context; evaluate the model and report methods and findings VC2M9A06	use mathematical modelling to solve applied problems involving inverse proportion, growth and decay, including in financial contexts to establish the compound interest formula as repeated applications of simple interest; formulate problems, choosing to apply linear, quadratic or exponential models; interpret solutions in terms of the situation; evaluate and modify models as necessary and report assumptions, methods and findings VC2M10A15

Source: https://victoriancurriculum.vcaa.vic.edu.au/vc-maths-v2/scope-and-sequence.html

Mapping the activity to content descriptions

Year 7

- recognise and use variables to represent everyday formulas algebraically and substitute values into formulas to determine an unknown VC2M7A01
- solve one-variable linear equations of increasing complexity with natural number solutions; verify equation solutions by substitution VC2M7A03
- investigate, interpret and describe relationships between variables represented in graphs of functions developed from authentic data VC2M7A04
- investigate, interpret and describe relationships between variables represented in graphs of functions developed from authentic data VC2M7A04

Year 8

- graph linear relations on the Cartesian plane using digital tools where appropriate; solve linear equations and one-variable inequalities using graphical and algebraic techniques; verify solutions by substitution VC2M8A02
- use algorithms and related testing procedures to identify and correct errors VC2M8A04
- experiment with linear functions and relations using digital tools, making and testing conjectures and generalising emerging patterns VC2M8A05

Mapping the activity to content descriptions(ctd)

Year 9

- sketch linear graphs of equations in various algebraic forms, using the coordinates of 2 points, and solve linear equations VC2M9A03
- find the gradient of a line segment, the midpoint of the line interval and the distance between 2 distinct points on the Cartesian plane VC2M9A04
- use mathematical modelling to solve applied problems involving change, including financial contexts involving simple interest; formulate problems, choosing to use either linear or quadratic functions or other simple variations; interpret solutions in terms of the context; evaluate the model and report methods and findings VC2M9A06

Year 10

- solve problems involving linear equations, including those derived from formulas VC2M10A07
- solve simultaneous linear equations, using algebraic and graphical techniques including using digital tools VC2M10A09
- construct scatterplots and consider a line of good fit; comment on the association between the 2 numerical variables in terms of strength, direction and linearity VC2M10ST02

Research and resource links

Research links

- Microsoft Word - RR_Volume4Done.doc (emis.de)
- (PDF) The effectiveness of geogebra when teaching linear functions using the IPad (researchgate.net)
- The balance model for teaching linear equations: a systematic literature review | International Journal of STEM Education | Full Text (springeropen.com)
- Learning to Graph Linear Functions: A Case Study of Conceptual Change: Cognition and Instruction: Vol 19, No 2 (tandfonline.com)
- Processes and reasoning in representations of linear functions | SpringerLink
- (PDF) Introducing Linear Functions: An Alternative Statistical Approach. (researchgate.net)

Resource links – Build Me Up Modules (MAV)

1. Rational Numbers (Fractions, Decimal Fractions, Percentages, Ratios and Proportional Reasoning)
2. Integer Operations (Order of Operations, Negative numbers, Squares and Square Roots, Exponents and Index Laws)
3. Algebra Foundations
4. Linear Functions
5. Measurement (including Angles and Logarithms)
6. Quadratics
7. Trigonometry
8. Space (including Networks)
9. Statistics Foundations
10. Statistics Applied
11. Probability Foundations
12. Probability Applied

https://www.mav.vic.edu.au/Resources/Build-Me-Up

Resource links

- Linear_equations (amsi.org.au)
- Year 8 Plotting linear relationships (amsi.org.au)
- Introduction_to_coordinate_geometry (amsi.org.au)
- What is Linear Function? - Equation, Graph, Definition (cuemath.com)
- Twopoint form calculator - with detailed explanation (mathportal.org)
- ms2-linear-book.pdf (adelaide.edu.au)
- Human Body Ratios - Scientific American
- Height and Bone Lengths (haifa.ac.il)
- Stature Estimation – FORENSICS (eforensics.info)
- 1.2.4 - Height Estimation from Bone: Human Body Systems - 604401 - West - Per 1, 2 (instructure.com)
- Mathematics | Tax, Super + You (taxsuperandyou.gov.au)
- calculating-amounts-to-be-withheld-from-13-October-2020.pdf (ato.gov.au)
- What should our maximum heart rate be during exercise? - ABC News
- Linear Functions – GeoGebra
- https://teacher.desmos.com/linear
- Linear Functions: An Introduction - FUSE - Department of Education & Training
- http://www.graphingstories.com/
- Match My Line • Activity Builder by Desmos

Feedback & accessing a copy of this PPT

I would welcome your thoughts on today's session.

This QR code will take you to a very short feedback form.

If you'd like a copy of the PPT for this session, you can either:

- include your email address when completing the feedback form, or
- email me directly: david@leighlancasterconsulting.com.au

Then I'll share a link over the next few days.

Feedback MAV

Please also complete the MAV feedback survey within the conference app.

Any quick questions?

Thank you

Leigh-Lancaster Consulting

https://leighlancasterconsulting.com.au